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Charles P. Dickerson, Mark W. Arndt,
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Evaluation of Vehicle Velocity Predictions Using the Critical Speed Formula

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ABSTRACT

Tire marks left by the vehicle prior to impact, rollover, or other event, are important forensic evidence reconstruction of motor vehicle accidents. Often these tire marks have some curvature that is measured and used to calculate the speed of vehicles prior to the event. This calculation is based on the coefficient of friction of the tire/road interface and the radius of curvature of the vehicle center of gravity (c.g.) path. There is controversy about the validity of this approach. To explore this theory, a test vehicle was driven through a series of maneuvers that produced yaw marks for direct comparison of actual vehicle velocity to the velocity calculated by the critical speed formula. Test results show the critical speed formula is inaccurate for most circumstances and does not correctly describe vehicle limit performance behavior.

INTRODUCTION

Tire yaw marks are any tire marks left on a roadway by a rolling tire of a turning (yawing) vehicle. These yaw marks are produced by lateral accelerations dependent on tire type and size, tire inflation, road surface, vehicle weight, suspension geometry, and vehicle maneuver. The yaw mark is a result of slipping of the tire contact patch as the tire generates the side force necessary to alter the course of the vehicle. Yaw marks appear when the side force requirements approach the tire's maximum capability. The tire is said to be saturated when the limit of tire capability is achieved.

The radius of the yaw mark is commonly used to calculate the velocity of the vehicle using a critical speed formula that is based on tire/surface coefficient of friction. While the theory is relatively simple, there is controversy concerning when and how to use this formula or whether it is valid.

This paper describes a test program that was performed to evaluate the validity of using yaw mark curvatures to determine vehicle velocity.

THEORY

The literature refers to speed estimates based upon a "critical" speed "yaw" mark that is produced by motor vehicle tires during certain pre-crash yawing movements. The "critical" speed is described as the point where a motor vehicle begins to enter the limit of tire-to-road surface adhesion (tire saturation) in a turn. In other words, the lateral force applied on the tire by the body of the vehicle exceeds the tire's ability to generate an equally opposing force at the contact patch.

The equation used to calculate velocity from tire mark curvature is derived from the equation for the centripetal acceleration for a particle on a circular path and is shown below:

$$v = \sqrt{f\ g\ R}$$

Where: f is the tire/road coefficient of friction
R is the path radius
g is gravitational acceleration

Legitimate questions arise as to whether the results from using this equation accurately calculates vehicle speed.

LITERATURE REVIEW

Calculating vehicle speed using the radius of curvature of a tire yaw mark produced during pre-crash yaw movements is described in numerous documents on motor vehicle traffic crash investigation and reconstruction.[1-4] This method is often taught in traffic crash reconstruction schools and has been the subject of numerous published discussions, validations, and criticisms.[5-9]

In general, all sources agree that specific information is required to estimate vehicle speed from tire yaw marks. The information[1,2] requirements are, a) the marks observed on the road must be yaw marks, b) the radius of the marks must be measurable, c) the road grade must be in the direction of slippage and, d) the roadway coefficient of friction must be obtained.

Most sources describe suspected limitations and/or conditions that may affect the validity of such yaw marks in analysis.

Because of the diverse circumstances producing tire yaw marks, various authors place conditions on the characteristics of yaw marks used in speed estimates. The character of striations in yaw marks is used as a criterion for selecting appropriate yaw marks from which to make speed estimates.[5] Several authors note that the particular nature of striation in yaw marks may relate to braking, acceleration, or vehicle slip angle.[4,6,8]

Criteria for selecting yaw marks for making speed estimates include examination for adequate lateral acceleration as demonstrate by sufficient yaw marking on the roadway[4] and position of rear tire yaw marks relative to the front tire yaw marks.[6] It is further suggested that the critical speed formula not be used in the event of excessive vehicle sideslip.[6]

Measurement of the tire yaw marks and the subsequent determination of a vehicle c.g. radius of curvature is commonly addressed. Universally, measurements of a chord and middle ordinate of the curve path are used to calculate the yaw mark radius. In some cases, the actual path of the center of gravity is discussed. [2,6]

There is considerable discussion in the literature concerning which part of the yaw mark to use. It is suggested that the early phase of the yaw mark be used. Recommendations for the chord length vary from "the first third of the mark" to 100 feet.[6,7]

The appropriate coefficient of friction is most often implied or stated to be the average or steady state acceleration as measured in straight line locked wheel braking.[7] The use of sliding friction values are noted to be appropriate.[6] Downward adjustments to coefficient of friction are suggested, depending upon the vehicle age[4] and the direction of yaw mark striations.[9]

Researchers have conflicting conclusions as to the speeds determined from tire yaw marks. Speed estimates have been described as being reasonable for determining minimum speeds[8] and as being useful only in placing an upper bound on speeds.[9]

Overall, the technical literature expresses special concerns for a variety of compounding variables, including tire pressure, tir construction, anti-lock brakes, changes in vehicl trim (steering, acceleration/braking), vehicle loading and weight, and even the competency and motivation of the investigator.[1,4,5,6]

TEST DESCRIPTION

The test vehicle used in this program was a 1988 Ford Escort GT equipped with P195/60R15 Goodyear Eagle GT+4 tires inflated to the vehicle recommended pressure. Ballast was secured in the right rear seat and luggage area to balance the vehicle laterally and move the center of gravity rearward. This was done to reduce vehicle understeer for increased slip angles. Five point restraints, roll bar, and outriggers were added to the vehicle for driver protection.

The vehicle was instrumented with a 14-channel onboard digital acquisition system to collect the data channels listed in Table 1. A paint marker gun and strobe light were installed and synchronized with the acquisition system for correlating electronic data, tire marks, and video.

Table 1: Instrumentation Channel List

Channel No.	Measurement
1	Right Rear Wheel Velocity
2	Yaw Angle
3	Pitch Angle
4	Roll Angle
5	Steering Angle
6	Left Front Wheel Angle
7	Right Front Wheel Angle
8	Longitudinal Acceleration @ c.g.
9	Lateral Acceleration @ c.g.
10	Vertical Acceleration @ c.g.
11	Yaw Angle
12	Roll Angle
13	Lateral Velocity @ c.g.
14	Longitudinal Velocity @ c.g.

Electronic surveying equipment was used to measure the tire marks. Sixty to eighty data points were collected for each mark. Additionally, all marks were photographed.

The test was performed on a concrete airport apron that provided a reasonably homogenous surface.

The maneuvers used were right circle turns, right step steers, and left/right double step steers. Maneuver severity was chosen to produce a range of lateral accelerations and slip angles.

A 30.48 meter (100 ft.) radius circle was used for the circle turn. The maneuver is performed by slowly accelerating the vehicle from a standing stop to maximum attainable speed around the circle. This was accomplished in approximately three quarters of a turn, and four segments of the resulting tire marks were measured and photographed. The vehicle exhibited understeer at the limit maneuver.

The step steer maneuvers consisted of driving the test vehicle to a prescribed speed and rapidly applying a right steer input and maintaining the steer input through completion of the maneuver. The throttle was released prior to the steer input. The speeds were between 88.5 kph (55 mph) and 104.6 kph (65 mph) and steer inputs were between 45 and 180 degrees.

The double-step steer maneuver was performed by applying a step steer to the left, pausing, then applying a step steer to the right. The purpose of this maneuver was to generate high slip angle tire marks. It was necessary to apply some light braking during the steer inputs. The braking level was just enough to generate oversteer without approaching wheel lockup. The brake input was removed once the vehicle began oversteering.

A straight line locked wheel stop from 60 mph was performed to measure the coefficient of friction of the tire/road interface. A 0.77 G's steady state value was achieved during the slide and was used in the calculation. Figure 1 shows the results of the locked wheel stop. A typical lateral acceleration curve from one of the double-step steer runs also is plotted.

It is interesting that the lateral acceleration level exceeds the locked wheel stop longitudinal acceleration. A transient lateral acceleration as high as 1.2 G's was observed early in the maneuver. Overall the steady state lateral acceleration was approximately 0.05 G's higher than the locked wheel stop longitudinal acceleration.

DATA ANALYSIS

Upon completion of each test maneuver, the tire yaw marks were measured using electronic survey equipment and the radii of the yaw marks calculated for various segments along the tire path. The test vehicle velocities were then calculated using the critical speed formula and compared to the actual measured velocities.

Two algorithms for calculating the path radius were considered. The first was the "ABC Method," which uses the distance between three arbitrary points on the curve to calculate radius. The second method, "Chord-Middle Ordinate Method,"[10] uses a chord length and the perpendicular distance from the center of the chord to the curve (middle ordinate). Of the two approaches, the Chord-Middle Ordinate approach proved to be less sensitive to signal noise in the yaw mark position data.

The radius of the measured segment was found by using the endpoints of each segment as the chord and calculating the middle ordinate in the circle test data. The left front tire mark was used in all of the calculations, and in some cases the left rear also was used. The measured instantaneous velocity at the center of the segment was compared to the predicted velocity data. The slip presented was calculated from the measured velocity data. The results from the circle turn data are shown in Table 2.

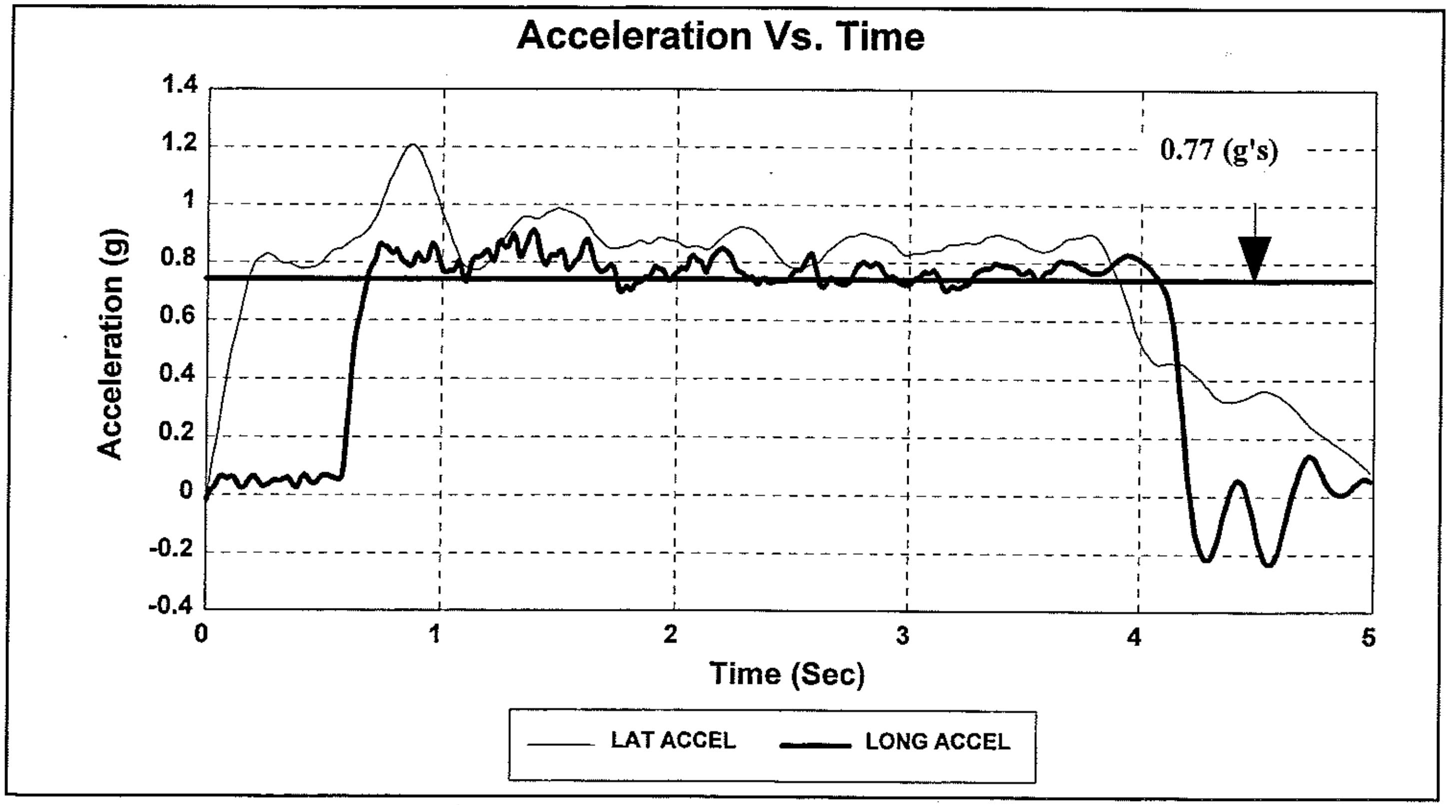

Figure 1: Acceleration Comparison

Table 2: Circle Turn Results

Segment No.	Path Radius m/ft	Predicted Velocity kph/mph	Measured Velocity kph/mph	Lateral Accel. (G)	Slip Angle (deg)	% Error
1.	31.1/102	55.2/34.3	46.5/28.9	.68	0.10	18.0
2.	33.5/110	57.3/35.6	49.1/30.5	.67	0.55	17.0
3.	31.7/104	55.7/34.6	52.0/32.3	.80	0.43	7.0
4.	35.7/117	59.1/36.7	53.9/33.5	.46	0.00	9.5

Table 3. Step Steer Results

Segment no.	Path Radius m/ft	Calculated Velocity kph/mph	Measured Velocity kph/mph	Lateral Accel. (G)	Slip Angle (deg)	% Error
1.	115.8/380	106.5/66.2	82.4/51.2	.50	1.3	29
2.	109.4/359	103.5/64.3	80.3/49.9	.52	1.0	29

Table 4. Step Steer Results

Segment no.	Path Radius m/ft	Calculated Velocity kph/mph	Measured Velocity kph/mph	Lateral Accel. (G)	Slip Angle (deg)	% Error
1.	72.3/237	84.2/52.3	82.54/51.3	.84	3.1	2
2.	65.8/216	80.3/49.9	72.6/45.1	.77	1.9	10

Table 5. Double-Step Steer Results

Segment no.	Path Radius m/ft	Calculated Velocity kph/mph	Measured Velocity kph/mph	Lateral Accel. (G)	Slip Angle (deg)	% Error
1.	120.4/395	108.6/67.5	95.6/59.4	.80	10.8	14
2.	103.9/341	100.9/62.7	92.0/57.2	1.0	23.8	10
3.	105.2/345	101.5/63.1	84.2/52.3	.91	36.2	21
4.	99.7/327	99.0/61.5	75.6/47.0	.92	45.0	31
5.	96.0/315	97.0/60.3	65.4/40.6	.84	57.1	48
6.	85.0/279	91.2/56.7	52.1/32.4	.83	61.0	75
7.	22.3/73	46.5/28.9	31.2/19.4	.85	50.5	49

Table 6. Double-Step Steer Results

Segment no.	Path Radius m/ft	Calculated Velocity kph/mph	Measured Velocity kph/mph	Lateral Accel. (G)	Slip Angle (deg)	% Error
1.	97.8/321	97.8/60.8	92.7/57.6	.82	10.7	6
2.	87.2/286	92.4/57.4	84.3/52.4	.89	25.1	10
3.	87.2/286	92.4/57.4	75.3/46.8	.76	27.0	23
4.	71.0/233	83.3/51.8	66.0/41.0	.66	21.0	27
5.	60.4/198	76.9/47.8	60.5/37.6	.35	2.5	27
6.	52.4/172	71.8/44.6	54.5/33.9	.53	1.7	32
7.	70.4/231	83.0/51.6	45.5/28.3	.29	0.2	82

Each of the tables represents the results from one test run. The segments are the portions of an individual tire mark that were measured and used in the calculations. The segments are ordered chronologically in the tables. Segment one is early in the maneuver and the last segment late in the maneuver. Figure 2 shows the vehicle path, orientation, and tire mark for the data shown in Table 5. Along the tire mark, the segments used for the velocity calculation are shown.

15.3 meters (50 ft) to 18.3 meters (60 ft) segments were used for the radius calculations of the step and double-step steer runs. The segment was applied as a moving window so that the middle of the segment became the endpoint for the next segment as the window traveled through the data. The instantaneous measured velocity and slip angle values from the center of the segment were used for the comparison. As with the circle data, the lateral acceleration was filtered using a 0.5 second moving average. Tables 3 through 6 present results from the step and double step steer runs.

DISCUSSION

Analysis of the results show two visible trends. Error goes up with slip angle, and up as

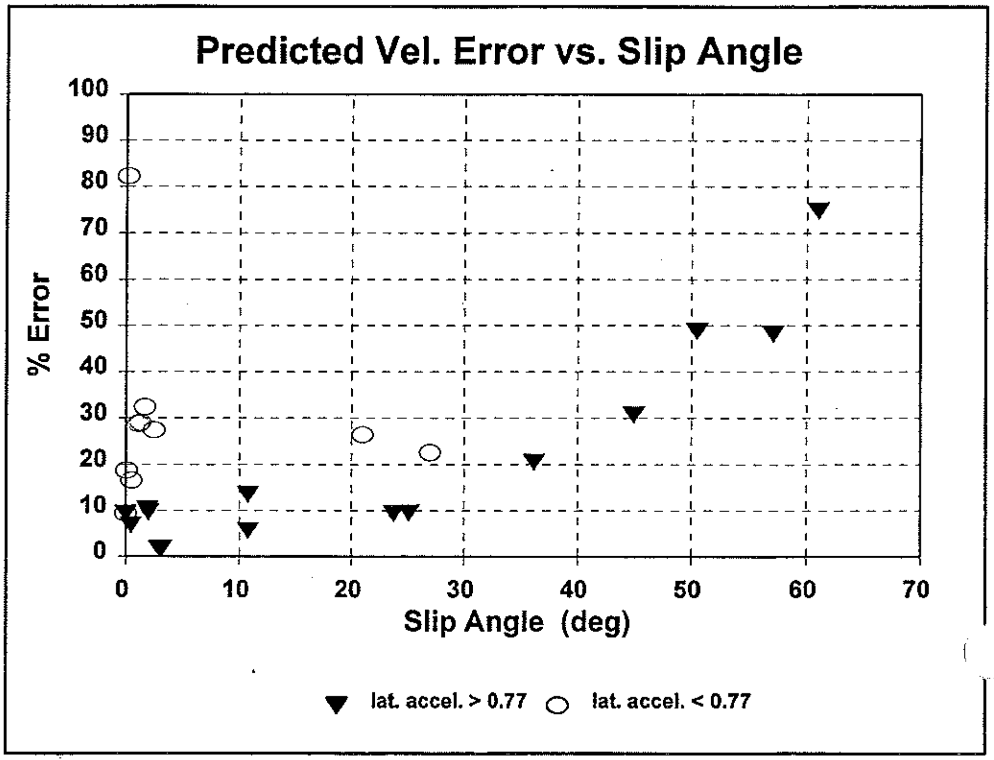

Figure 3: Error vs. Slip Angle

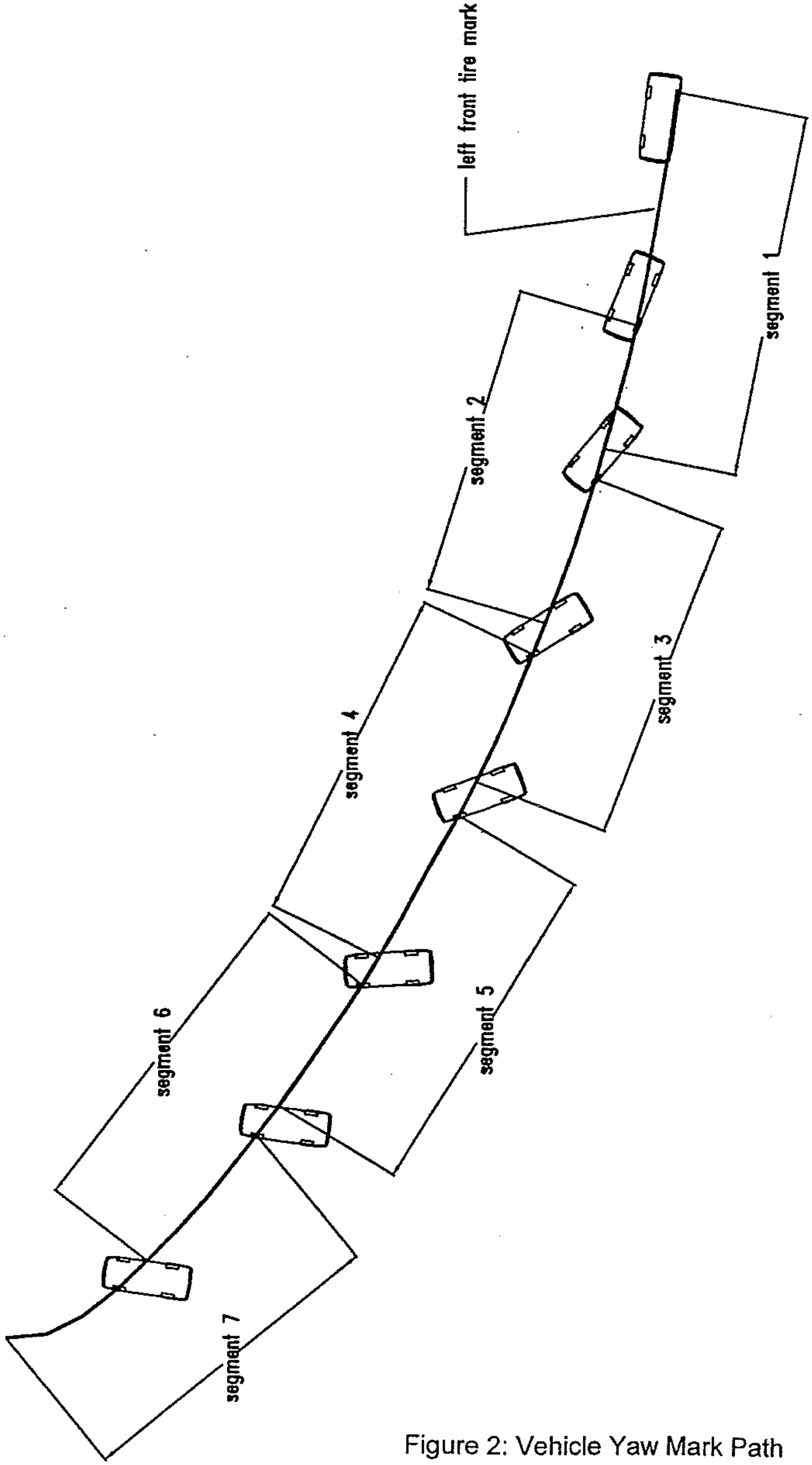

Figure 2: Vehicle Yaw Mark Path

lateral acceleration goes down.

The calculated velocity error as a function of slip angle for the circle turn, step steer and double-step steer maneuvers are shown in Figure 3. Calculated velocity error is presented in the far right column of the tables above and is defined as the difference between the calculated velocity and the measured velocity divided by the measured velocity. Data with lateral acceleration below 0.77 G is distinguished from data where lateral acceleration is above 0.77 G.

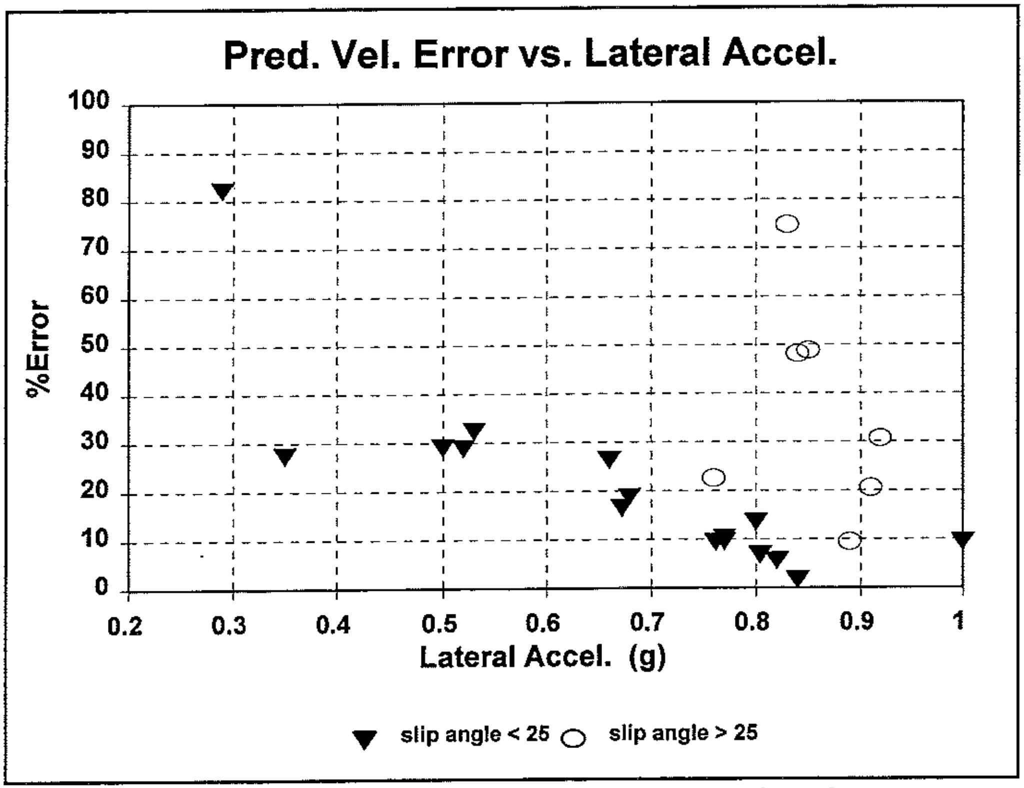

Figure 4: Percent Error vs. Lateral Acceleration

The lowest errors occur when slip angles are below 25 degrees and lateral accelerations at or above the measured tire/road surface coefficient of friction (0.77 G's). Typically, the calculation is approximately 10% high, with the exception of one calculation which is a 2% error.

Figure 4 shows the calculated velocity error as a function of the lateral acceleration in circle turn, step steer and double step steer maneuvers. Data with slip angles above and below 25 degrees are distinguished. The error curve drops to approximately 10% near 0.77 G's with the lowest errors being above 0.8 G's.

Results in the tables indicate that the best calculated speed results occur in the test runs when lateral acceleration is above 0.8 G's and the slip angle is less than 25 degrees. 2% errors were calculated in the run that exhibited high lateral acceleration (>0.8 G's) and low slip angles (approx. 3 degrees) throughout the entire run. In runs where large slip angles (>25 degrees) occurred, errors of 6% to 14% were calculated in the early part of the run while slip angles were approximately ten degrees. This suggests that for maneuvers which result in high slip angles, the method is inaccurate even early in the maneuver.

Calculated velocities using the critical speed formula were greater than the measured velocities throughout the testing. The error in the calculated speed occurs when slip angle is high or lateral acceleration is low. The actual path radius increases in the event of high slip angle or below maximum lateral acceleration due to maximum tire side force not being generated at the tire/road interface. This results in a calculated speed that is greater than the actual speed.

SUMMARY

A test vehicle was driven through a series of maneuvers to produce yaw marks for direct comparison of actual vehicle velocity to the velocity calculated by the critical speed formula. The critical speed formula calculations were greater than the measured velocities of the test vehicle in all test results.

The data demonstrates that the approach is most accurate under a very specific set of vehicle conditions:

1. The vehicle is at its limit cornering capability.
2. Slip angle is less than 20 to 25 degrees throughout the maneuver.

Even when these conditions were met, the error in the predicted values was approximately 10%.

This test program reveals that the critical speed formula is too simple to accurately describe the behavior of vehicles that leave yaw marks on the roadway.

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